I think most of you have problem with HW2.57(a). Here, I am going to do mathematical derivation.

2.57 A sequence has the DTFT

$$X\left(e^{jw}\right) = \frac{1-a^2}{\left(1-ae^{-jw}\right)\left(1-ae^{jw}\right)}, |a| < 1$$

(a) Find the sequence x[n].
(b) Calculate $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})cos(w)dw$

(a) First of all, we need to multiply numerator and denominator by $(-\frac{1}{a}e^{-jw})$

$$X\left(e^{jw}\right) = \frac{\left(1-a^2\right)\left(-\frac{1}{a}e^{-jw}\right)}{\left(1-ae^{-jw}\right)\left(1-ae^{jw}\right)\left(-\frac{1}{a}e^{-jw}\right)} \tag{1}$$

$$= \frac{\left(\frac{-1}{a}+a\right)e^{-jw}}{\left(1-ae^{-jw}\right)\left(1-\frac{1}{a}e^{-jw}\right)} \tag{2}$$

We assume that,

$$X\left(e^{jw}\right) = \frac{A}{\left(1-ae^{-jw}\right)} + \frac{B}{\left(1-\frac{1}{a}e^{-jw}\right)} \quad = \frac{A\left(1-\frac{1}{a}e^{-jw}\right)+B\left(1-ae^{-jw}\right)}{\left(1-ae^{-jw}\right)\left(1-\frac{1}{a}e^{-jw}\right)} \tag{3}$$

Hence, we get

$$A + B = 1$$
$$-A\frac{1}{a} - Ba = a - \frac{1}{a}$$

Consequently, we get, $A = 1$ and $B = -1$ and,

$$X\left(e^{jw}\right) = \frac{1}{\left(1-ae^{-jw}\right)} + \frac{-1}{\left(1-\frac{1}{a}e^{-jw}\right)}$$

From the problem description, we know that $|a| < 1$ so $|\frac{1}{a}| > 1$. For stable system, we know that the ROC contains unit circle. It means that,

$$\frac{1}{\left(1-ae^{-jw}\right)}, ROC : |z| > |a|$$

$$\frac{-1}{\left(1-\frac{1}{a}e^{-jw}\right)}, ROC : |z| < |a|$$

Therefore, the sequence $x[n]$ is,

$$x[n] = a^n u(n) + a^{-n}u(-n-1) = a^{|n|}$$

Note that, the anti-causal system means that the outputs and internal states depend solely on the future input values. example,

$$h[n] = -\alpha^n u(-n-1)$$

$$H(z) = \frac{1}{1-\alpha z^{-1}}, ROC : |z| < \alpha$$

Also, the ROC for anti-causal system contains 0.

For causal system, it means that the outputs and internal states depend on past input values.

$$h[n] = \alpha^n u(n)$$

$$H(z) = \frac{1}{1-\alpha z^{-1}}, ROC : |z| > \alpha$$

The ROC for causal system contains ∞.

(b) want to get $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})cos(w)dw$ From Eular's Identity,

$$cos(w) = \frac{e^{jw} + e^{-jw}}{2}$$

$$\begin{aligned}
\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})cos(w)dw &= \frac{1}{2}\left[\int_{-\pi}^{\pi} X(e^{jw})e^{jw}dw + \int_{-\pi}^{\pi} X(e^{jw})e^{-jw}dw\right] && (4)\\
&= \frac{1}{2}\left[x(-1) + x(1)\right] && (5)\\
&= \frac{1}{2}\left[a^{|-1|} + a^{|1|}\right] && (6)\\
&= a && (7)
\end{aligned}$$

The solution from text book is wrong. Please be ware that the inverse fourier transform is a scalar value instead of a sequence.