Depth Reconstruction from Sparse Samples: Representation, Algorithm, and Sampling (Supplementary Material)

Lee-Kang Liu, Student Member, IEEE, Stanley H. Chan, Member, IEEE and Truong Q. Nguyen, Fellow, IEEE

Abstract

The purpose of this supplementary report is to present experimental results on the selecting parameters for the proposed depth reconstruction algorithm.

I. DEPTH RECONSTRUCTION ALGORITHM

For completeness we first recall the reconstruction algorithm. Referring to the main article, the optimization problem that we would like to solve is

$$\text{minimize} \quad \frac{1}{2} \| Sx - b \|^2_2 + \sum_{\ell=1}^{L} \lambda_\ell \| W_\ell \Phi_\ell^T x \|_1 + \beta \| x \|_{TV},$$

where $\Phi_\ell$ is a dictionary, $W_\ell$ is the corresponding weighting matrix, $S$ is the sampling pattern, and the vector $b$ is the observation. We consider $L = 2$ dictionaries, where $\Phi_1$ and $\Phi_2$ are the wavelet and contourlet dictionaries, respectively.

To solve (1), we apply the alternating direction method of multipliers (ADMM). The ADMM algorithm tries to find the stationary point of the augmented Lagrangian function, defined as

$$L(x, u_1, u_2, r, v, w, y_1, y_2, z) = \frac{1}{2} \| b - Sr \|^2 + \lambda_1 \| W_1 u_1 \|_1 + \lambda_2 \| W_2 u_2 \|_1 + \beta \| v \|_1$$

$$- w^T (r - x) - y_1^T (u_1 - \Phi_1^T x) - y_2^T (u_2 - \Phi_2^T x) - z^T (v - Dx)$$

$$+ \frac{\mu}{2} \| r - x \|^2 + \frac{\rho_1}{2} \| u_1 - \Phi_1^T x \|^2 + \frac{\rho_2}{2} \| u_2 - \Phi_2^T x \|^2 + \frac{\gamma}{2} \| v - Dx \|^2.$$

Note that in (2), we need to specify the regularization parameters ($\lambda_1, \lambda_2, \beta$) and internal parameters ($\mu, \rho_1, \rho_2, \gamma$). The purpose of this supplementary report is to discuss how the parameters are chosen.

II. PARAMETER SELECTION AND EXPERIMENTS

We now present how to choose the regularization parameters ($\lambda_1, \lambda_2, \beta$), and the internal parameters ($\mu, \rho_1, \rho_2, \gamma$).

A. Experimental Configurations

Before presenting results, we first describe our experimental configurations. Testing disparity maps are chosen from Middlebury datasets. All disparity values are normalized to the range $[0, 1]$. Figure 1 shows some examples of disparity maps. For the sampling patterns, we choose the uniformly random samples to minimize any bias towards the sampling. For wavelet dictionary, we use “db2” wavelet function with decomposition level 2, and for contourlet dictionary, we set frequency partition “5, 6”. These settings are fixed throughout the experiment.

Fig. 1: Example disparity maps from Middlebury dataset.

1http://vision.middlebury.edu/stereo/data/
B. Regularization Parameters ($\lambda_1, \lambda_2, \beta$)

We empirically evaluate the mean square error (MSE) by sweeping the parameters ($\lambda_1, \lambda_2, \beta$) from $10^{-6}$ to $10^0$, with a fixed sampling rate of $\xi = 0.2$. The optimal values of the parameters are chosen to minimize the average MSE.

Figure 2 shows the MSE curves for various images. For each plot, the MSE is computed by sweeping one parameter while fixing the other parameters. Observing the top row of (2), we see that the optimal $\lambda_1$ across all images is approximately located in the range of $10^{-6} \leq \lambda_1 \leq 10^{-3}$. Therefore, we select $\lambda_1 = 4 \times 10^{-5}$. Similarly, we can determine $\lambda_2 = 2 \times 10^{-4}$ and $\beta = 2 \times 10^{-3}$.

We repeat the above analysis for $\xi = 0.1$. The results are shown in the bottom row of Figure 2. The result indicates that while there are some difference in the MSE as compared to the top row, the optimal value does not change. Therefore, we keep the parameters using the above settings.

![MSE curves for various images](image)

**Fig. 2:** Comparison of reconstruction performance with varying regularization parameters and depth images. For each plot, we sweep a parameter from $10^{-6}$ to $10^0$ while fixing others to be our typical values. We set the sampling rate to be 20% (1st row) and 10% (2nd row). Typical values of regularization parameters are $\lambda_1 = 4 \times 10^{-5}$, $\lambda_2 = 2 \times 10^{-4}$ and $\beta = 2 \times 10^{-3}$. 
C. Internal Parameters ($\mu, \rho_1, \rho_2, \gamma$)

For $\mu, \rho_1, \rho_2, \gamma$, we conduct a set of similar experiments as before. The results are shown in Figure 3 and Figure 4. The criteria to select the parameter is based on the convergence rate. This gives us $\rho_1 = 0.001, \rho_2 = 0.001, \mu = 0.01$ and $\gamma = 0.1$.

![Fig. 3: MSE for $\xi = 0.2$.](image-url)
Fig. 4: MSE for $\xi = 0.1$. 
III. SUMMARY

We summarize our findings in Table I. We remark that the values in Table I are “typical” values that correspond to a reasonable MSE on average. Of course, for a specific problem there exists a set of optimal parameters. However, from our experience, this set of parameters seems to be robust over a wide range of problems.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Functionality</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>Wavelet sparsity</td>
<td>$4 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Contourlet sparsity</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Total variation</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Half quad. penalty for Wavelet</td>
<td>0.001</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Half quad. penalty for Contourlet</td>
<td>0.001</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Half quad. penalty for $r = x$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Half quad. penalty for $v = Dx$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

TABLE I: Summary of Parameters and typical values.